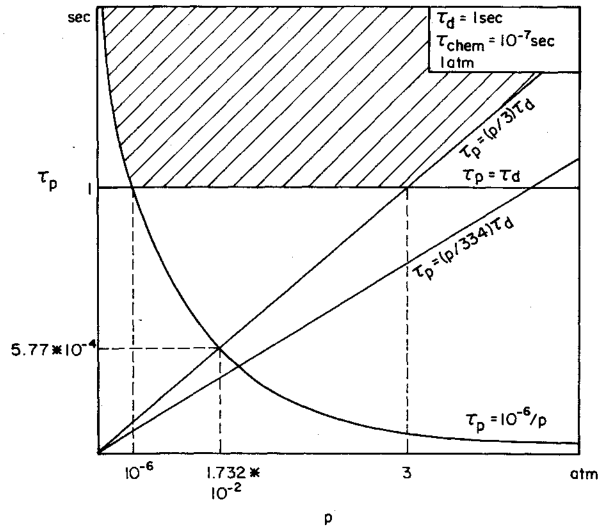

Fig. 1 Sketch representing the domain of validity for the quasi-steady gas field assumption.

assuming that the relationship $\tilde{R}/C_{pg}=(\gamma-1)/\gamma$ holds. Therefore,

$$\frac{p(0)}{T(0)\rho_g C_{pg}} \sim \frac{0.4}{1.4}\frac{10}{1} \sim 3 \tag{12}$$

Then Eq. (11) yields

$$\tau_p >> \tau_d/112 \tag{13}$$

There is no pressure dependence in this relationship.

b)
$$\frac{p(0)}{T(0)\rho_g C_{pg}} << \frac{\lambda_g}{\rho_g C_{pg}}\frac{1}{\beta R^2} \tag{14}$$

By use of Eqs. (12) and (6) one finds

$$\tau_p >> \tau_d \tag{15}$$

and no pressure dependence.

c)
$$\frac{p(0)}{T(0)\rho_g C_{pg}} << \frac{k_f}{w_F}\frac{C}{\beta C_{pg}T(0)}\frac{p(0)}{\tilde{R}(0)T(0)}\frac{P}{\theta}\frac{\tilde{R}(0)}{R} \tag{16}$$

Combining Eqs. (12) and (9) yields

$$\tau_p >> 30\tau_{\text{chem}} \tag{17}$$

without pressure dependence.

If Eqs. (4), (6), (9), (13), (15), and (17) are satisfied by the nonsteady problem, then the nonsteady situation can be treated as quasi-steady.

Figure 1 shows how one can find a region of quasi-steady behavior in the (p, τ_p) plane for a given droplet of a fixed size. The figure is based upon the previous six inequalities, and the numerical evaluations were obtained for

$$\tau_{\text{chem}_{/\text{atm}}} = 10^{-7}\text{sec and } \tau_d = 1 \text{ sec}$$

corresponding to droplets of 1-mm diam.[3] The domain in which the quasi-steady assumption is acceptable is covered with oblique lines in Fig. 1. This region is quite large and is certainly in the range of interest for the low pressure range in Diesel engines. For the purpose of applying this theory to combustion in engines or rockets, τ_d should be taken much smaller because in these cases the diameter of the droplets is at least one order of magnitude smaller than 1 mm.[1] As the value of τ_d decreases, the domain of validity for the quasi-steady assumption becomes larger.

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Flutter of Thermally Buckled Finite Element Panels

T. Y. Yang* and A. D. Han†

Purdue University, West Lafayette, Ind.

Introduction

THE versatile finite element method has recently been extended to the panel flutter problem with noteworthy success. Olson[1,2] first developed the concept of aerodynamic matrix and formulated several plate finite elements for the flutter analysis of rectangular and delta wings. Similar work was conducted by Kariappa[3,4] et al. In those developments, the linearized supersonic flow theory approximated by the first-order frequency of oscillation was used. Yang[5] later used a numerical integration technique to incorporate in the finite element formulation an exact linearized flow theory that includes the higher-order frequencies.

In this study, the finite element formulation given in Ref. 1 is extended to treat the flutter problem of a semi-infinite panel which is buckled into large deflections due to aerodynamic heating. The postbuckling behavior of the panel is predicted by a piecewise linear incremental procedure[6] combined with coordinate transformation at every step. The results obtained from illustrative examples are compared with alternative solutions[7-9] with reasonable agreement.

Formulation

The semi-infinite panel is defined by the following parameters: chord length a; thickness h; mass density m (mass per unit area); bending rigidity $D=Eh^3/12(1-\nu^2)$; nondimensional deflection $W=w/a$; and thermal expansion coefficient α. One surface of the panel is exposed to a supersonic flow with velocity U, Mach number M, and mass density ρ (mass per unit volume). The flow thus has a pressure $q=1/2\,\rho U^2$.

The panel is supported at both edges with the trailing edge free to roll in the plane of the panel. The trailing edge is also restrained elastically in the plane direction of the panel with elastic constant k (force per unit spanwise length). The panel is assumed to be subjected to uniform aerodynamic heating with a temperature rise of ΔT degrees. The heating produces inplane compressive stress N_x (force per unit spanwise length) and causes buckling when $N_x=\pi^2 D/a^2$. The rise in tem-

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*Professor, School of Aeronautics and Astronautics. Associate Fellow AIAA.

†Formerly Graduate Student. Presently Engineer, Aeronautical Research Lab., Taiwan, Republic of China.

perature required to buckle the panel is

$$\Delta T_{cr} = \pi^2 D / E\beta\alpha h a^2 \tag{1}$$

where β is the effective stiffness coefficient of the support at the trailing edge,

$$\beta = 1/(1 + Eh/ak) \tag{2}$$

It is known that a temperature rise of 5° to 10°F is sufficient to produce buckling in many panels. After the buckling has taken place, the inplane compressive stress is no longer merely a linear function of ΔT, it is also a function of the trailing edge displacement that is a quadratic function of slopes. It has the form

$$R_x = N_x a^2/D = (Eha^2\beta/D)\left[\alpha\Delta T - \frac{1}{2}\int_0^1 (\partial W/\partial\xi)^2 d\xi\right] \tag{3}$$

where $\xi = x/a$ is the nondimensional distance measured from the leading edge.

Assuming two-dimensional supersonic aerodynamic theory and considering the effect of the flow on only one surface of the panel, the equation of motion for the buckled, initially flat, semi-infinite panel becomes

$$\frac{\partial^4 W}{\partial\xi^4} + \lambda\frac{\partial W}{\partial\xi} + \lambda\frac{M^2-2}{M^2-1}\frac{a}{U}\frac{\partial W}{\partial t} + \frac{ma^4}{D}\frac{\partial^2 W}{\partial t^2} - R_x\frac{\partial^2 W}{\partial\xi^2} = 0 \tag{4}$$

where $\lambda = 2qa^3/D\sqrt{M^2-1}$ is the nondimensional aerodynamic velocity or pressure. It is noted that Eq. (4) is valid only when $M > 1.6$. Equation (4) is nonlinear because the term R_x is a quadratic function of the displacement derivatives.

A finite element formulation for Eq. (4) can be written as

$$\{F\} = \Big[[K] + [M] + [A] + [N]\Big]\{Q\} \tag{5}$$

where the stiffness, mass, and aerodynamic matrices $[K]$, $[M]$, and $[A]$ for a four degree-of-freedom (a deflection and a slope at each edge) semi-infinite plate element was formulated by Olson.[1] The incremental stiffness matrix $[N]$ due to the effect of inplane force R_x can be found in Ref. 6. Since R_x is a quadratic function of the edge displacement $\{Q\}$, Eq. (5) is nonlinear.

The nonlinear equation (5) is solved by a step-by-step linear incremental procedure. A coordinate transformation procedure is required at every step to modify the four matrices.[6] The explicit forms for the 6×6 transformed matrices $[K]$ and $[N]$ were given in Ref. 6. The same procedure is used to transform the matrices $[M]$ and $[A]$. For the first step of the postbuckling prediction, a small displacement vector conforming to the eigenvector for the Euler buckling eigenvalue is assumed to initiate the postbuckling.

$$\frac{1}{2}\int_0^1\left(\frac{\partial W}{\partial\xi}\right)^2 d\xi = \frac{1}{2}\begin{Bmatrix} W_1 \\ (\partial W/\partial\xi)_1 \\ W_2 \\ (\partial W/\partial\xi)_2 \end{Bmatrix}^T \begin{bmatrix} 6/5 & & & \text{symmetric} \\ -1/10 & 2/15 & & \\ -6/5 & 1/10 & 6/5 & \\ -1/10 & -1/30 & 1/10 & 2/15 \end{bmatrix} \begin{Bmatrix} W_1 \\ (\partial W/\partial\xi)_1 \\ W_2 \\ (\partial W/\partial\xi)_2 \end{Bmatrix} \tag{6}$$

Results

The first example considered is a simply-supported buckled panel modeled by four finite elements. The results obtained for the relations among the nondimensional midchord deflection, stream velocity, and temperature rise are shown in Fig. 1. This example was solved previously by Houbolt (Ref. 7 or 8) using the Galerkin's modal method. By retaining only the first two modes, the intricate nonlinear equations of motion could be simplified and his results are also shown in Fig. 1. For a two-mode representation, the first mode component for amplitude is the midchord deflection. Fair agreement between the two solutions is shown in Fig. 1.

In Fig. 1, each curve represents the postbuckling behavior of the panel for a specific value of stream velocity λ. At $\lambda = 0$, the example represents a well-known elastica postbuckling problem and its solution can be obtained exactly in an elliptical integral form. By setting $k = \infty$, it was found that the present finite element solution agrees with the exact solution with the maximum discrepancy in deflection less than one percent. From such verification, it may be concluded that the present results for nonzero λ's should be more accurate than the Galerkin's two-mode solutions. It may also be concluded that the relation between the square of the deflection (W^2) and the temperature rise is not linear as those obtained by two-mode Galerkin's method.

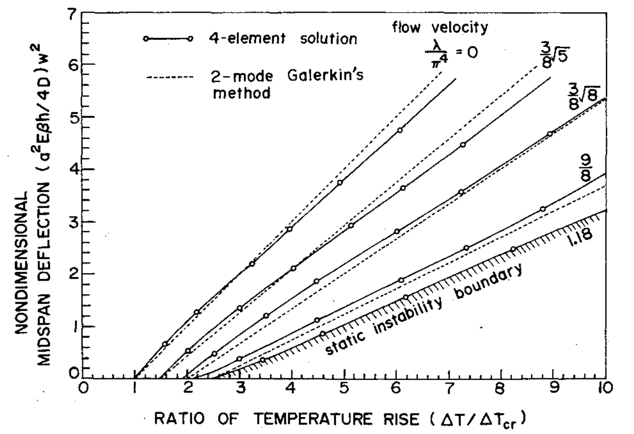

Fig. 1 Relation among the midspan deflection, flow velocity, and temperature rise for a simply-supported and semi-infinite buckled panel.

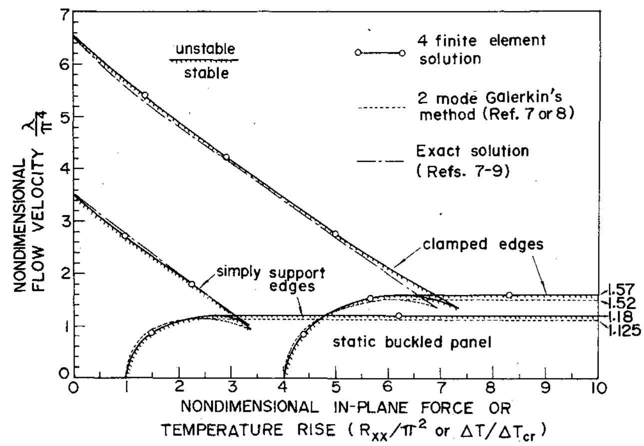

Fig. 2 The static and dynamic stability boundaries of flat and buckled panels.

During the computation of the results in Fig. 1, each velocity λ was first included in the aerodynamic matrix which was subsequently combined with the stiffness matrix, the Euler buckling load was then calculated with the neglect of the mass matrix, the postbuckling behavior was finally predicted by the piecewise linear incremental procedure.

In Fig. 1, it is seen that in the region where $\Delta T/\Delta T_{cr} < 1$, no buckling deflection will occur. In the region where $1 > \Delta T/\Delta T_{cr} < 2.485$, an increase in the velocity λ will increase the pressure of the airstream which tends to stabilize or blow flat the buckled panel. In the region where $\Delta T/\Delta T_{cr} > 2.485$, an increase in velocity λ will reduce the buckled depth until it reaches the value of $1.180\pi^4$. When the velocity reaches beyond this value, the static instability will occur. Mathematically speaking, when $\lambda > 1.180\pi^4$ the eigenvalue for the Euler buckling temperature rise ΔT becomes imaginary whereas a real solution is not obtainable.

The stability boundaries for the simply-supported buckled panels can be presented in a simpler form by using the data obtained in Fig. 1 with the neglect of the deflection parameter. The boundary is shown as the lower curve in Fig. 2. The case of clamped edge condition is also considered and the results are also plotted in Fig. 2. Both curves are slightly higher than the ones obtained by the Galerkin's two-mode approximation. It is seen in Fig. 2 that as long as the velocity stays below the horizontal line for each panel, the buckled panel will always be stable.

An observation of the results in Fig. 2 reveals that in order for dynamic instability to occur for $\Delta T/\Delta T_{cr}$ less than 2.485 and 6.161 for the simply-supported and clamped panels, respectively, the panels must be blown flat or unbuckled. The dynamic instability boundaries for various values of temperature rise ΔT were obtained and shown as the two upper curves in Fig. 2. Such curves were found by first specifying the value of ΔT and then varying the value of flow velocity λ in small increment until the eigenvalue for first mode frequency changes from real to complex. It is seen that these results are in close agreement with the exact solutions.[7-9]

Conclusions

The basic procedure for the flutter analysis of flat finite element panels with elastic boundary constraints and subjected to temperature rise is outlined. The aerodynamic theory is based on the piston theory with first-order approximation. The Mach number considered is limited to be beyond approximately 1.6. With the use of a linear incremental procedure combined with a coordinate transformation technique, the postbuckling behavior of a heated panel under the stabilizing effect of airstream pressure can be predicted and the static instability boundary can be found. The dynamic instability boundary for the flat panel can also be found.

This basic procedure can easily be extended to the general panel system for practical application in the aeronautical engineering industry. The general cases can be rectangular panels of finite aspect ratio; panels of delta or other arbitrary shapes; panels with cut-outs; complex elastic edge conditions; stiffened and composite panels; and slightly curved panels, etc. All these panels can be subjected to the aerodynamic heating.

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Theory of Adjoint Structures

H. Van Belle*
GTE (ATEA), Herentals, Belgium

Nomenclature

$|A|'$ = transpose of $|A|$
$|B|'$ = complex conjugate of $|B|$
$|C|^*$ = complex tranjugate of $|C|$

I. Introduction

MECHANICAL structures usually are designed in an iterative way. The design procedure starts with an "as good as possible" intuitive design of the structure. Then, the structure is analyzed to check if the design specifications are met. If this is not so, the design must be improved and analyzed again. This cycle is repeated until the specifications are met in the best way. The automatic design of a mechanical structure can be reduced to the minimizing of a performance function. The performance function is a measure of the deviation between the actual and desired behavior of the structure. To decrease the number of iteration steps it is possible to use an optimization strategy. In many important optimization methods, the most effective improvements are derived from the gradients of the performance function.[1] These gradients or sensitivities give the influence of the building element parameters on the performance function of the whole mechanical structure.

In the classical methods of sensitivity analysis it is assumed that the structures are linear and statically or kinematically determinate.[2-6] This supposition makes the calculation of the sensitivities very easy. The stiffness matrix of a linear structure, for instance, is equal to

$$|K'| = |B|'|K||B| \tag{1}$$

where

$|K|$ = stiffness matrix of the building elements, considered together but not connected
$|B|$ = compatibility conditions

If the structure is kinematically determinate, the sensitivity of the stiffness matrix for the change of a parameter R_k of a building element is given by

$$\partial|K'|/\partial R_k = |B|'(\partial|K|/\partial R_k)|B| \tag{2}$$

Indeed, the compatibility conditions and the matrix $|B|$ are not influenced by the parameters R_k. This is not the case for

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*Assistant Process Engineering Manager, Process Engineering Dept.